

Charmed  $B(B_s)$  decays involving a light tensor meson in PQCD approach

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We study the  $B(B_s) \rightarrow D_{(s)}(\bar{D}_{(s)})T$  and  $D_{(s)}^*(\bar{D}_{(s)}^*)T$  decays in perturbative QCD approach, where T denotes a light tensor meson. In addition to the branching ratios, we also give predictions to the polarization fractions for the  $D_{(s)}^*(\bar{D}_{(s)}^*)T$  decays. For those decays with a tensor meson emitted, the factorizable emission diagrams do not contribute because of lorentz covariance considerations. In order to give the predictions to these decay channels, it is necessary to go beyond the naive factorization to calculate the nonfactorizable and annihilation diagrams.

## I. INTRODUCTION

Recently, the measurements about B decays involving a light tensor meson (T) have been making progress [1]. For the tensor meson considered in this work,  $J^P = 2^+$ , the quark pair's orbital angular momentum L and total spin S are equal to 1. However, their production property in B decays is quite similar to the light vector mesons, because the polarizations with  $\lambda = \pm 2$  do not contribute due to the angular momentum conservation [2]. These rare B decays have been studied in the naive factorization [3, 4]. In this work, we consider these B decays, which have been discussed in the factorization approach [5], involving a heavy D meson and a tensor meson in the final states. Due to the fact that  $\langle 0 | j^\mu | T \rangle = 0$ , where  $j^\mu$  is the  $(V \pm A)$  or  $(S \pm P)$  current [3, 6], there is no contribution from the factorizable diagrams with a tensor meson emitted. Because of this difficulty, the naive factorization approach can not give the right prediction. So we must go beyond the naive factorization to calculate the nonfactorizable contributions and the annihilation type contributions. In this paper, we use the perturbative QCD factorization approach (PQCD) [7] to calculate these contributions

There are various energy scales involved in hadronic  $B(B_s)$  decays, i.e., three different scales: W boson mass scale, B meson mass scale  $M_B$  and the factorization scale  $\sqrt{\Lambda M_B}$ , where  $\bar{\Lambda} \equiv M_B - m_b$ . The electroweak physics higher than W boson mass can be calculated perturbatively. By using the renormalization group equation, the physics between B meson mass scale and W boson mass scale can be included in the Wilson coefficients of the effective four-quark operators. The hard part in the PQCD approach includes the physics between  $M_B$  and the factorization scale. The physics below the factorization scale is nonperturbative, which can be described by the universal hadronic wave functions of mesons. In PQCD approach, in order to avoid the endpoint singularity which spoil the perturbative calculation, we do not neglect the transverse momentum  $k_T$  of the light quarks in meson. As a result, the additional energy scale will appear, which is introduced by the transverse momentum. The additional scale can give large double logarithms in the hard part calculations. We can resum these double logarithms by using the renormalization group equation to give a Sudakov factor, which effectively suppresses the end-point region contribution and makes the PQCD approach more reliable and consistent.

## II. FORMALISM

We know that the light quark in  $B(B_s)$  meson is soft, because the heavy b quark carries most of the energy of  $B(B_s)$  meson. But it is collinear in the final state light meson, so a hard gluon is necessary to connect the spectator quark to the four quark operator. The hard part of the interaction in PQCD approach contains six quarks rather than four quarks, which is called six-quark effective theory or six-quark operator. The decay amplitude can be explicitly factorized as the following formalism:

$$\mathcal{A} \sim \int d[x] b_1 db_1 b_2 db_2 b_3 db_3 \times \text{Tr} \left[ C(t) \Phi_B(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right], \quad (1)$$

TABLE I: Branching ratios (unit:  $10^{-7}$ ) and the percentage of transverse polarizations  $R_T$  (unit: %) of  $B_{(s)} \rightarrow D^* T$  decays calculated in the PQCD approach together with results from ISGW II model [9, 10].

Decay Modes	Class	Br(PQCD)	Br(SDV [9])	Br(KLO [10])	$R_T$
$B^0 \rightarrow D^{*0} a_2^0$	C	$1.34^{+0.64+0.25+0.17}_{-0.53-0.19-0.15}$	0.50	...	$47^{+3.7+1.4}_{-4.5-1.6}$
$B^0 \rightarrow D^{*0} f_2$	C	$2.70^{+1.22+0.43+0.36}_{-1.02-0.30-0.33}$	0.53	...	$26^{+3.7+1.3}_{-4.0-1.1}$
$B^0 \rightarrow D^{*0} f_2'$	C	$0.052^{+0.023+0.008+0.007}_{-0.02-0.005-0.006}$	0.01	...	$26^{+3.7+1.3}_{-4.0-1.1}$
$B^0 \rightarrow D^{*0} K_2^{*0}$	C	$60.5^{+25.3+10.6+8.30}_{-21.3-9.15-7.56}$	19	18	$22^{+2.7+1.0}_{-3.2-0.5}$

where  $b_i$  is the conjugate variable of transverse momentum  $k_{iT}$ .  $x_i$  is the momentum fractions in mesons.  $t$  is the largest energy scale in calculation of  $H(x_i, b_i, t)$ .  $C(t)$  are the corresponding Wilson coefficients of four quark operators.  $\Phi(x)$  are the meson wave functions.  $S_t(x_i)$  is obtained by the threshold resummation, that smears the end-point singularities on  $x_i$ . The  $e^{-S(t)}$  term is the so-called Sudakov form factor. It can effectively suppresses the soft dynamics and the long distance contributions in the large  $b$  region.

The polarization tensor  $\epsilon_{\mu\nu}(\lambda)$  with helicity  $\lambda$  of tensor meson can be expanded via the vectors  $\epsilon^\mu(0)$  and  $\epsilon^\mu(\pm 1)$ , which are the polarization vector of vector meson, [6]

$$\begin{aligned}
\epsilon^{\mu\nu}(\pm 2) &\equiv \epsilon(\pm 1)^\mu \epsilon(\pm 1)^\nu, \\
\epsilon^{\mu\nu}(\pm 1) &\equiv \sqrt{\frac{1}{2}} [\epsilon(\pm 1)^\mu \epsilon(0)^\nu + \epsilon(0)^\mu \epsilon(\pm 1)^\nu], \\
\epsilon^{\mu\nu}(0) &\equiv \sqrt{\frac{1}{6}} [\epsilon(+1)^\mu \epsilon(-1)^\nu + \epsilon(-1)^\mu \epsilon(+1)^\nu] + \sqrt{\frac{2}{3}} \epsilon(0)^\mu \epsilon(0)^\nu.
\end{aligned} \tag{2}$$

Due to the angular momentum conservation argument, the  $\pm 2$  polarizations do not contribute in these considered decays [2]. This can effectively simplify the following perturbative calculations. What is more, the light-cone distribution amplitudes (LCDAs) of the tensor meson are antisymmetric under the interchange of momentum fractions of the quark and anti-quark in the SU(3) limit [6], due to the Bose statistics.

### III. RESULTS AND DISCUSSION

The numerical results of these considered decay branching ratios and polarization fractions are displayed in Ref.[8]. There are no direct CP asymmetries, because these decays do not have contributions from the penguin operators, and we find that compared with  $B \rightarrow D^{(*)} T$  decays, the  $B \rightarrow \bar{D}^{(*)} T$  decays are enhanced by the CKM matrix elements  $|V_{cb}/V_{ub}|^2$ .

In table I, one can see that for the color suppressed (C) decay modes, the predicted branching ratios in the PQCD approach are larger than those of Ref.[9] and Ref.[10]. For example,  $B^0 \rightarrow \bar{D}^0 f_2$ , our predicted branching ratio is  $\mathcal{B}(B^0 \rightarrow \bar{D}^0 f_2) = 9.46 \times 10^{-5}$ . It is larger than other approaches, but agrees better with the experimental data  $(12 \pm 4) \times 10^{-5}$  [1]. For these color suppressed decay channels, the non-factorizable contributions play dominant role in the amplitude, because the factorizable contribution is suppressed by the Wilson coefficient  $a_2$  ( $C_1 + C_2/3$ )  $\simeq 0.1$ , while the Wilson coefficient for nonfactorizable contribution is  $C_2 \simeq 1.0$ . What is more, when the emitted meson is the  $D(\bar{D})$  meson or tensor meson, the contributions for two nonfactorizable emission diagrams no longer cancel with each other, because of the big difference between the  $c$  quark and the light quark in  $D$  meson and the antisymmetry of the tensor meson wave function.

From table II, we can find that for those color allowed (T) decay channels, our predicted branching ratios basically agree with that predicted in naive factorization approach in Ref.[9]. The dominant difference is mainly caused by parameter changes and the additional contributions from nonfactorizable and annihilation diagrams. For some color allowed decays, for example,  $B^0 \rightarrow D^- a_2^+$ , there are no contributions from factorizable emission diagrams, because

TABLE II: Branching ratios (unit: $10^{-7}$ ) and the percentage of transverse polarizations  $R_T$ (unit:%) of  $B_{(s)} \rightarrow D^* T$  decays calculated in the PQCD approach together with results from ISGW II model [9, 10].

Decay Modes	Class	Br(PQCD)	Br(SDV [9])	Br(KLO [10])	$R_T$
$B^+ \rightarrow D^{*+} f_2'$	T	$0.29^{+0.15+0.03+0.02}_{-0.11-0.04-0.02}$	0.21	...	$25^{+1.3+1.8}_{-1.0-1.8}$
$B_s \rightarrow D^{*+} K_2^{*-}$	T	$14.8^{+7.42+0.90+1.94}_{-5.93-0.85-1.77}$	12	...	$26^{+1.2-0.1}_{-1.0-0.2}$

TABLE III: Branching ratios (unit: $10^{-5}$ ) and the percentage of transverse polarizations  $R_T$ (unit:%) of  $B_{(s)} \rightarrow \bar{D}^* T$  decays calculated in the PQCD approach together with results from ISGW II model [9, 10].

Decay Modes	Class	Br(PQCD)	Br(SDV [9])	Br(KLO [10])	$R_T$
$B^0 \rightarrow \bar{D}^{*0} a_2^0$	C	$39.3^{+13.6+2.05+2.15}_{-11.1-0.50-1.34}$	12	7.8	$73^{+4.6+9.0}_{-4.3-8.1}$
$B^0 \rightarrow \bar{D}^{*0} f_2$	C	$38.2^{+13.9+1.97+2.10}_{-11.6-1.22-1.30}$	13	8.4	$70^{+5.9+9.4}_{-5.6-6.3}$
$B^0 \rightarrow \bar{D}^{*0} f_2'$	C	$0.72^{+0.26+0.02+0.04}_{-0.22-0.03-0.03}$	0.26	0.11	$70^{+5.9+9.4}_{-5.6-6.3}$
$B^0 \rightarrow \bar{D}^{*0} K_2^{*0}$	C	$5.32^{+1.69+0.79+0.32}_{-1.42-0.66-0.22}$	1.3	1.1	$71^{+1.8+8.6}_{-1.6-8.8}$

the emitted meson is the tensor meson  $a_2^+$ . For these decays, we give the predictions for the first time by calculating the contributions from non-factorizable diagrams and the annihilation type diagrams.

According to the power counting rules in the factorization assumption, from the quark helicity analysis, the longitudinal polarization should be dominant [11, 12]. However, in table III, for those color suppressed (C)  $B \rightarrow \bar{D}^* T$  decays with the  $\bar{D}^*$  emitted, the transverse polarization fractions are about 70%. For these decays, the  $\bar{c}$  quark and the  $u$  quark in  $\bar{D}^*$  meson are all produced through  $(V - A)$  current, and the  $\bar{c}$  is right-handed; while the  $u$  quark is left-handed. As a result, the  $\bar{D}^*$  meson is longitudinally polarized. We know that the helicity of massive quark can flip easily. So the  $\bar{c}$  quark can flip easily from right handed to left handed, then the polarization of the  $\bar{D}^*$  meson can be equal to -1. On the other hand, because of the additional contribution of orbital angular momentum, the recoiled tensor meson can also be transversely polarized with polarization  $\lambda = -1$ . Thus, the transversely polarized contributions are no longer small. In table I, for color suppressed (C)  $B \rightarrow D^* T$  decays with  $D^*$  meson emitted, the percentage of transverse polarizations are only at the range of 20% to 30%. After the similar analysis, we know that the emitted  $D^*$  meson can also be transversely polarized with the polarization  $\lambda = +1$ . According to the angular momentum conservation, the recoiled tensor meson must be also transversely polarized with polarization  $\lambda = +1$ . In this case, the tensor meson needs contributions from both orbital angular momentum and spin, so the situation is symmetric. As we know that the wave function of tensor meson is asymmetric. Therefore the transversely polarized contribution is suppressed, because of Bose statistics.

We also find very large transverse polarizations up to 80% for the W annihilation (A) type  $B \rightarrow D^* T$  decays in table IV. The light quark and anti-quark produced through hard gluon are left-handed or right-handed with equal opportunity. So the  $D^*$  meson can be longitudinally polarized, and also be transversely polarized with polarization  $\lambda = -1$ . For the tensor meson, the anti-quark from weak interaction is right-handed; while the quark produced from hard gluon can be either left-handed or right-handed. When taking into account the additional contribution from the orbital angular momentum, the tensor meson can be longitudinally polarized or transversely polarized with polarization  $\lambda = -1$ . So the transverse polarization can become so large with additional interference from other diagrams.

#### IV. SUMMARY

We investigate the  $B_{(s)} \rightarrow D^{(*)} T, \bar{D}^{(*)} T$  decays within the framework of perturbative QCD approach. We find that the nonfactorizable and annihilation type diagrams play important roles in the amplitude, especially for those color suppressed channels and the decays with a tensor meson emitted. For the decays with a tensor meson emitted, we

TABLE IV: Branching ratios (unit: $10^{-7}$ ) and the percentage of transverse polarizations  $R_T$ (unit:%) of  $B_{(s)} \rightarrow D^*T$  decays calculated in the PQCD approach together with results from ISGW II model [9, 10].

Decay Modes	Class	Br(PQCD)	Br(SDV [9])	Br(KLO [10])	$R_T$
$B^+ \rightarrow D^{*+} K_2^{*0}$	A	$18.2^{+4.77+0.21+2.00}_{-5.15-2.15-2.70}$	...	...	$82^{+2.1+3.8}_{-2.9-2.7}$
$B^+ \rightarrow D_s^{*+} f_2'$	A	$21.6^{+6.77+1.00+3.00}_{-6.03-2.32-2.70}$	4.0	2.0	$83^{+5.2+1.9}_{-5.3-1.9}$
$B^+ \rightarrow D_s^{*+} \bar{K}_2^{*0}$	A	$1.25^{+0.36+0.06+0.16}_{-0.34-0.16-0.15}$	...	...	$81^{+1.6+3.7}_{-1.8-3.3}$

give the predictions for the first time. For those color suppressed  $B_{(s)} \rightarrow \bar{D}^*T$  decays, the nonfactorizable diagrams provide sizable transversely polarized contributions. For those W annihilation type  $B \rightarrow D^*T$  decays, the factorizable annihilation diagrams give large transverse polarized contributions up to 80%.

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